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SPECIAL FEATURES OF THE TEMPERATURE MODE OF STEEL PARTS IN SURFACE-STRENGTHENING GRINDING

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A mathematical model of heat transfer in steel parts in force grinding is considered. The model allows one to determine parameters of grinding at which the layer of steel of the given thickness is heated to the temperatures of quenching. The main laws governing the changes in the temperature of the part being ground are presented. The situation of the appearance of cleavages and melting of the edge of a wedge-like body in grinding and sharpening is analyzed.

Intense heat release in grinding greatly influences the quality of the forming surfaces of parts. Here, of special importance is not only the value of the contact temperature but also the spatial-temporal temperature field. Use of energy feasibilities of grinding for simultaneous strengthening of the treated surface is very urgent and in some cases it allows one to exclude operations of heat treatment with the required geometric accuracy being provided. However, wide adoption of the method of strengthening grinding (SG) [1, 2] is restrained by insufficient studies of intense thermal fields, which are formed under the action of the modes of treatment and exert a considerable effect on the degree and depth of the strengthened zone. A majority of the known mathematical models of grinding presuppose calculation of contact temperatures [3, 4]. Some authors present relations which allow the determination of the character of the distribution of these temperatures over the depth of the part being ground [5, 6]. At the same time, the relations mentioned have a very approximate character and do not suit for calculation of the parameters of the process of strengthening grinding. Moreover, they do not explain the appearance of cleavages and melts of the edge of the part being ground. By virtue of this fact we suggest calculating the parameters of the process of grinding by a more accurate mathematical model based on a direct solution of the nonstationary equation of heat conduction.

Mathematical Model for Determining Temperature Parameters of the SG Process. The mathematical model for determining temperature fields in the material subject to the action of an abrasive disk is based on solution of the well-known equation of heat conduction which with the assumption of heat supply only from the surface of the material has the form [7]

$$c_p(T)\,\rho(T)\,\frac{\partial T(\mathbf{r},\tau)}{\partial\tau} + \nabla\left(-\lambda(T)\,\nabla T(\mathbf{r},\tau)\right) = 0\,. \tag{1}$$

The boundary conditions to Eq. (1) which allow for convective and radiative mechanisms of heat transfer from the surface of the body and heat supply due to forces of cutting and friction on the surface of the material can be written in the form

$$-\lambda(T)\,\frac{\partial T(\mathbf{r},\tau)}{\partial \mathbf{n}}\bigg|_{\mathbf{r}\in B} = \alpha(T)\,(T(\mathbf{r},\tau) - T_{\mathrm{m}}) + \varepsilon\sigma\,(T^4(\mathbf{r},\tau) - T_{\mathrm{m}}^4) + q_{\mathrm{gr}}(\mathbf{r},\tau)\,. \tag{2}$$

At the initial instant of time the temperature of the part is taken to be uniform

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$$T(\mathrm{r}, \tau)\,|_{\tau=0} = T_0 \,. \tag{3}$$

The temperature of the part $T(\mathrm{r}, \tau)$ is found from solution of the equation of heat conduction (1) with boundary (2) and initial conditions (3). The mentioned system is solved by the method of finite elements [8, 9]. In accordance with this method spatial digitization of the calculation region is made, as a result of which a number of elements of splitting (N_e) and nodes (N_n), where the temperature is calculated, are obtained. For each i-th node ($1 \le i \le N_\mathrm{n}$) we introduce the basis functions $\psi_i(\mathrm{r})$ so that $\psi_i(\mathrm{r}_i) = 1$ and $\psi_i(\mathrm{r}_j) = 0$, $\forall_j \ne i$, where $1 \le j \le N_\mathrm{n}$, i.e., the function ψ_i is a "hyperpyramid" constructed above the i-th node. Then, the temperature $T(\mathrm{r}, \tau)$ is expressed by the basis functions as

$$T(\mathrm{r}, \tau) = \sum_{j=1}^{N_\mathrm{n}} T_j(\tau)\, \psi_j(\mathrm{r}) \,. \tag{4}$$

To determine the temperature at the nodal points of the calculation region T_j we used the Bubnov–Galerkin method [10], in accordance with which for each $1 \le i \le N_\mathrm{n}$ from Eq. (1) we have

$$\iiint_W \left(c_p(T)\, \rho(T) \frac{\partial T(\mathrm{r}, \tau)}{\partial \tau} + \nabla\left(-\lambda(T)\, \nabla T(\mathrm{r}, \tau)\right)\right) \psi_i(\mathrm{r})\, dW = 0 \,, \tag{5}$$

where $W = \bigcup_{e=1}^{N_e} \Delta_e$ is the calculation region; Δ_e is the e-th element of splitting.

With account for (4) expression (5) is reduced to the system of linear equations

$$\sum_{j=1}^{N_\mathrm{n}} \frac{\partial T_j(\tau)}{\partial \tau} \sum_{e=1}^{N_\mathrm{e}} \iiint_{\Delta_e} \left(c_p(T)\, \rho(T)\, \psi_i(\mathrm{r})\, \psi_j(\mathrm{r})\right) dW +$$

$$+ \sum_{j=1}^{N_\mathrm{n}} T_j(\tau) \sum_{e=1}^{N_\mathrm{e}} \left(\iiint_{\Delta_e} \lambda(T)\, \nabla\psi_i(\mathrm{r})\, \nabla\psi_j(\mathrm{r})\, dW + \iint_{\delta\Delta_e} \psi_i(\mathrm{r})\, \psi_j(\mathrm{r})\, \tilde{\alpha}(T)\, dS \right) =$$

$$= \sum_{e=1}^{N_\mathrm{e}} \iint_{\delta\Delta_e} \psi_i(\mathrm{r})\, \tilde{q}(T)\, dS \quad (1 \le i \le N_\mathrm{n}) \,, \tag{6}$$

where $\delta\Delta_e$ is the edge of the e-th element pertaining to the boundary of the body S, and the quantities $\tilde{\alpha}(T)$ and $\tilde{q}(T)$ with account for boundary conditions (2) are determined as

$$\tilde{\alpha}(T, \mathrm{r}) = \alpha(T, \mathrm{r}) + \varepsilon\sigma\left(T(\mathrm{r})^3 + T(\mathrm{r})^2\, T_\mathrm{m} + T(\mathrm{r})\, T_\mathrm{m}^2 + T_\mathrm{m}^3\right) ,$$

$$\tilde{q}(T, \mathrm{r}, \tau) = \tilde{\alpha}(T, \mathrm{r}, \tau)\, T_\mathrm{m} + q_\mathrm{gr}(\mathrm{r}, \tau) \,. \tag{7}$$

For further consideration it is convenient to write the system of equations (6) in the matrix form

$$\mathbf{M}^C(\tau) \frac{\partial \mathbf{T}(\tau)}{\partial \tau} + \mathbf{M}^\lambda(\tau)\, \mathbf{T}(\tau) = \mathbf{Y}(\tau) \,, \tag{8}$$

where $\mathbf{T}(\tau)$ and $\mathbf{Y}(\tau)$ are the vectors of the quantities on the right side of system (6) and the temperature at the nodes of the computational grid. Equation (8) is solved by the Crank–Nicolson [10] difference scheme

$$\hat{\mathbf{M}}^C(\tau) \frac{\mathbf{T}(\tau^{k+1}) - \mathbf{T}(\tau^k)}{\Delta\tau} + \frac{1}{2} \hat{\mathbf{M}}^\lambda(\tau) \left(\mathbf{T}(\tau^{k+1}) + \mathbf{T}(\tau^k)\right) = \hat{\mathbf{Y}}(\tau) \,, \tag{9}$$

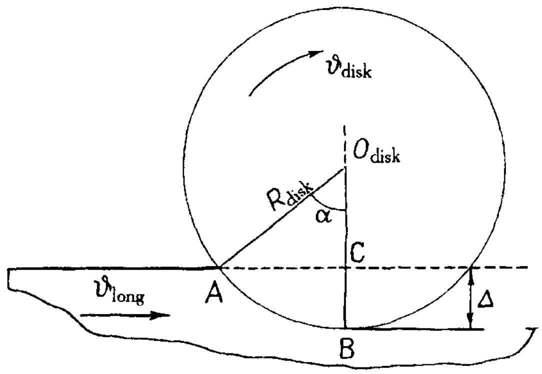


Fig. 1. Schematic of calculation of the length of contact between the disk and the part in grinding of a flat surface.

where $\Delta\tau$ is the time step; the superscript "$\hat{}$ "indicates a mean value of some quantity during the time interval $[\tau^k, \tau^{k+1}]$. With account for (9) we can write the expression for determining $\mathbf{T}(\tau^{k+1})$

$$\mathbf{T}(\tau^{k+1}) = (\mathbf{M}_\tau)^{-1} [\mathbf{M}_{p.d} \mathbf{T}(\tau^k) + \mathbf{Y}_{p.d}], \tag{10}$$

where $\mathbf{M}_\tau = \widehat{\mathbf{M}}^C - (\Delta\tau/2)\widehat{\mathbf{M}}^\lambda$; $\mathbf{M}_{p.d} = \mathbf{M}^C + (\Delta\tau/2)\widehat{\mathbf{M}}^\lambda$; $\mathbf{Y}_{p.d} = \Delta\tau\widehat{\mathbf{Y}}$.

With account for the fact that the obtained matrices $\mathbf{M}_\tau$ and $\mathbf{M}_{p.d}$ are band or positive definite, it is convenient to inverse the matrix $\mathbf{M}_\tau$ in (10) by the Kholesskii method, which requires less computer resources for storage and solution of matrices.

With account for that said above, the algorithm of calculation of temperature fields $T(\mathbf{r}, \tau)$ can be presented as:

1) we set an initial temperature of the part T_0, $\tau = 0$;

2) we determine the following instant of time $\tau^{k+1} = \tau^k + \Delta\tau$;

3) we determine the surface density of the heat source caused by the forces of cutting and friction $q_{gr}(\mathbf{r}, \tau)$ at the boundary nodes of the computational grid and the coefficients $\widetilde{\alpha}(T)$ and $\widetilde{q}(T)$;

4) in accordance with the above-described method of the solution of the equation of heat conduction (1) and according to (6) we fill the matrices $\mathbf{M}_\tau$ and $\mathbf{M}_{p.d}$ and the vector of the right side $\mathbf{Y}$. Then, according to (10) we calculate the values of the temperature at the nodes of the computational grid $T_i(\tau^{k+1})$;

5) if the time of calculation is less than that assigned, we repeat calculation from item 2.

The described algorithm allows one to calculate a temperature field in the part subject to grinding at the known value of the heat source – the surface density of the heat source $q_{gr}(\mathbf{r}, \tau)$ caused by forces of cutting and friction.

To find the density of the heat flux, we consider the schemes of motion of the abrasive disk along the surface of the part (Fig. 1).

An additional heat flux $q_{gr}(\mathbf{r}, \tau)$ appears only at the place of contact between the abrasive disk and the part (arc AB) and it can be expressed by the relation

$$q_{gr}(\mathbf{r}) = \left[\frac{F_{cut}\vartheta_{disk}}{LB} - q_{disk} - q_{met}\right] \delta(\mathbf{r}, L, B), \tag{11}$$

where

$$\delta = \begin{cases} 1, & \mathbf{r} \in [\text{contact zone}], \\ 0, & \mathbf{r} \notin [\text{contact zone}]. \end{cases}$$

As is shown in [6], q_{disk} does not exceed several percent of the heat released due to the forces of cutting and friction, i.e., we can assume $q_{disk} = 0$. The heat flux escaping with the taken-off metal is

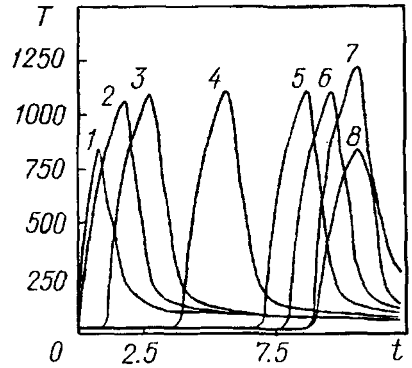


Fig. 2. Change of temperature at control points of the surface of the specimen in grinding.

$$q_{\text{met}} = \frac{c_p M}{LB} (T - T_0) , \tag{12}$$

where $M = \rho \Delta \vartheta_{\text{long}} B$ is the expenditure of taken-off metal per second, kg/sec.

At present, the force of cutting F_{cut} is determined basically from experiment. It is written in the form of empirical relations of the type [6]

$$F_{\text{cut}} = A \Delta^a S^b \vartheta_{\text{long}}^c \vartheta_{\text{disk}}^d , \tag{13}$$

where A, a, b, c, d are empirically selected coefficients; S is the area of the zone of contact between the disk and the part. The experiments show that within the range of velocities of the disk periphery 15–35 m/sec the force of cutting decreases with an increase in the velocity and $d \approx -0.33$ [6]. Then, it follows from physical considerations that the force of cutting is in direct proportion to the width of the disk covering (B) because with an increase in the width more and more new grains become operative. This cannot be said about the effect of the length of the contact zone (L), because, in spite of the increase in the number of working grains, a situation is possible where grains get into the grooves already cut by previous grains. However, in the first approximation we can assume that $F_{\text{cut}} \sim L$, and the coefficient A includes a correction for the length of the contact zone. With account for that said above, we have

$$F_{\text{cut}} = A \Delta^a \vartheta_{\text{long}}^c \vartheta_{\text{disk}}^{-0.33} LB . \tag{14}$$

We consider the length of the contact between the disk and the metal in surface grinding (Fig. 1):

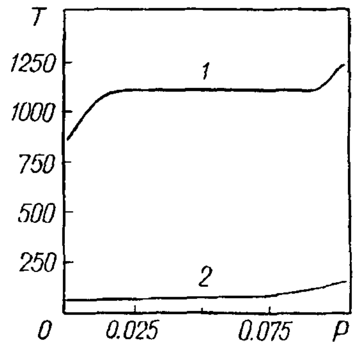
$$L = |AB| = R_{\text{disk}} \arccos \left(1 - \frac{\Delta}{R_{\text{disk}}} \right) \approx \sqrt{2 R_{\text{disk}} \Delta} . \tag{15}$$

Then $F_{\text{cut}} \sim \Delta^{a+0.5}$ and $F_{\text{cut}} \sim \vartheta_{\text{long}}^c$. In [6] an empirical relation is given according to which the force of cutting $F_{\text{cut}} \sim \Delta^{0.62}$ and $F_{\text{cut}} \sim \vartheta_{\text{long}}^{0.31}$. Thus, $a \approx 0.12$ and $c \approx 0.31$. With account for that said above, for the surface density of the heat flux in grinding we can suggest the following formula:

$$q_{\text{gr}} (\text{r}) = \left[A \Delta^{0.12} \vartheta_{\text{long}}^{0.31} \vartheta_{\text{disk}}^{0.67} - \frac{\rho \Delta \vartheta_{\text{long}} c_p}{L} (T - T_0) \right] \delta (\text{r}, L, B) . \tag{16}$$

As is shown by our experiments $A \approx 1.40 \cdot 10^7$.

Results of Numerical Simulation of the Thermal Mode of the Plate and the Wedge-Like Body in Surface-Strengthening Grinding. In grinding a flat part (with a section of 200 × 10 mm) the temperature of the surface at the initial instant of time is lower due to heat removal from the depth and streamwise, and on approaching the edge of the plate the temperature increases – the end effect of the termination of grinding becomes manifest. The



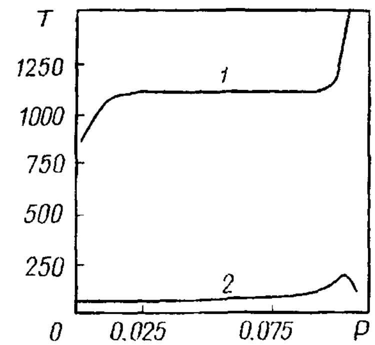


Fig. 3. Temperature of the surface as a function of the coordinate of the treated boundary in grinding at a constant frequency of disk rotation.

Fig. 4. Maximum temperature on the surface in grinding (1) and on cooling

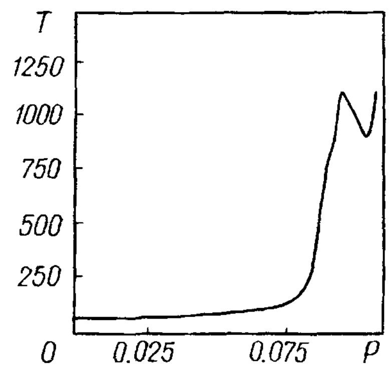


Fig. 5. Distribution of temperature on the surface of the wedge on termination of grinding.

control points 2, 3 in Fig. 2 are fixed with an interval of 10 mm from the point 1 at which the process of grinding is initiated, point 4 at the center of the part, and points 5, 6 with the same interval from point 7 at the boundary of the specimen, the end effect of the start of grinding disappears even at 20 mm from the start of grinding (point 3), the temperature of the central part of the plate remains constant (point 6 at 10 mm from the end of the specimen), and then the effect of the termination of grinding appears, which is characterized by a sharp increase in temperature.

At the given parameters of grinding the temperature at a depth of 1 mm reaches that of quenching (and with controlled cooling we can obtain the required hardness), but on reaching 2 mm from the surface (point 8) it decreases to about 800°C.

That said above allows us to draw the conclusion that in order to obtain a uniform temperature of the surface in strengthening grinding of flat parts it is necessary, in our opinion, to control the velocity of rotation of the disk – to increase it at the start of grinding, to maintain it constant in the middle, and to decrease on approaching the second edge.

This is most vividly characterized by the graph of the variation of the maximum temperature depending on the length of the section of a flat specimen (curve 1 in Fig. 3). The change in the temperature along the length of the specimen on termination of grinding is shown by curve 2,

Of special interest is the graph of the temperature of the surface in grinding of a wedge-like specimen. A distinctive feature is the sharp increase in the temperature at the end of the process by about 4-5 times compared to flat bodies, thus leading to bending and sometimes to melting of the sharp edge of the wedge (Fig. 4, curve 1).

The graph of the variation of the temperature on termination of the process of grinding is given in Fig. 4 (curve 2). The center of the wedge-like portion is the most heated; then the temperature decreases due to intense cooling of the developed surface of the wedge edge.

The result obtained when the abrasive disk approaches the sharp edge of the wedge (Fig. 5) is of interest. Due to the small thickness of the specimen the temperature on the edge begins to increase quickly and reaches a maximum much earlier than in the thick portion of the wedge due to removal of heat to the depth of the metal. With subsequent movement of the disk the deflection flattens, but the temperature of the edge of the wedge will exceed the admissible and, as a consequence, the edge of the wedge will melt.

Thus, we can draw the conclusion that in grinding a wedge-like part it is necessary to control the parameters that reduce the supply of heat to the body of the product. The most probable and effective parameter of this control can be the velocity of rotation of the disk or the velocity of disk delivery to the part.

NOTATION

$T(\mathbf{r}, \tau)$, the temperature of the body at the point with radius-vector r at the instant of time τ, $^{\circ}$C; c_p, ρ, λ, heat capacity, density, and coefficient of thermal conductivity of the material, respectively; n, outer normal to the boundary; ε, emissivity of the surface; $\sigma \approx 5.67 \cdot 10^{-8}$ W/(m$^2 \cdot$K^4), Stefan–Boltzman constant; α, coefficient of heat transfer in the system "part surface–surrounding medium", W/(m$^2 \cdot {}^{\circ}$C); T_{m}, temperature of the cooling medium; $q_{\mathrm{gr}}(\mathbf{r}, \tau)$, density of the heat flux caused by the forces of cutting and friction on the surface of the material in grinding; Δ, depth of grinding (taken-off allowance), m; $\vartheta_{\mathrm{long}}$, velocity of longitudinal delivery of the part, m/sec; R_{disk}, radius of the abrasive disk, m; $\vartheta_{\mathrm{disk}}$, velocity of motion of the periphery of the disk ($\vartheta_{\mathrm{disk}} = 2\pi R_{\mathrm{disk}} n$, where n is the frequency of rotation of the disk), m/sec; F_{cut}, force of cutting, N; L, length of contact of the disk and the part, m; B, width of the cutting edge of the disk, m; q_{disk}, heat flux escaping to the abrasive disk; q_{met}, heat flux escaping with the taken-off metal; P, coordinate of the treated boundary of the body, starting from the left, m.

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